REVIEWS

Supplement to the Seventy-fifth Annual Report of the Registrar-General of Births, Deaths, and Marriages, in England and Wales. Part II. Abridged Life Tables. London. 1920. Pp. xlvi–65.

The subject of this supplement is the preparation of abridged life tables. Short methods are explained and illustrated, showing how to calculate the various functions of a life table to a degree of accuracy closely approximating the values obtained by the more elaborate and extended methods accepted by actuaries. It is stated that "an abridged life table of sufficient accuracy for the purposes of a Medical Officer of Health may be constructed in three or four hours, provided ordinary death-rates in various age-groups of the population concerned are available." Numerous contributions on this subject have been published in statistical and actuarial journals, but this is the first official government publication in English of which I am aware that is devoted entirely to the subject of computing abridged life tables.

In Part I of the supplement George King gives a very satisfactory solution of this problem and illustrates his theory by a concrete example. His method or obvious extensions thereof would seem to be sufficient to meet the requirements of those whose mathematical attainments are limited but who nevertheless would like to prepare abridged life tables. King's method is one whereby the abridged life table values, like those in the extended table, are calculated from the original statistics without reference to the values in any other table.

The method explained in Part II of the supplement is interesting because it proceeds along entirely different lines. It is presented by E. C. Snow, M.A., D.Sc., and appears on pages v to xxv of the supplement under the title, "An Elementary Rapid Method of Constructing an Abridged Life Table." By an abridged life table is understood a life table in which the values of the mortality functions are not given for every single year of age, but only for a sufficient number of age intervals to serve the practical requirements of registrars and of health officers. In the present supplement the age intervals for the abridged life tables are 0–1, 1–2, 2–5, 5–10, 10–15, 15–20, 20–25, 25–35, 35–45, 45–55, 55–65, 65–75, 75–85, 85–95, 95–. The underlying assumption in Dr. Snow's method is that the probability that a person aged x will live n years, ${}_np_x$, can be expressed as a linear or quadratic function of the observed death rate, r. This relation or equation varies, not with the age group, but with different sections of the usual range of values covered by the death-rate, r. These equations are obtained by the solution, by the method of least squares, of a series of equations of the type

$$p=a+br,\ p=c+dr+er^2.$$

The values of r are observed death rates, and the corresponding values of p (or ${}_np_x$) are taken from certain arbitrarily selected life tables already calculated by some extended method. For example, when $x>10$ and $.00300 \leqq r \geqq .00370$, the

quadratic relation for five-year groups is

$$_5p_x = .98152 + 5095.5(.00383 - r)^2,$$

and when $.00500 \leq r \leq .00800$, the quadratic relation for ten-year groups is

$$_{10}p_x = .07286 + 26.8859(.18575 - r)^2.$$

The proof offered for the validity of these assumptions other than that they seem reasonable is the close agreement between the values obtained by the application of the method to those actually calculated by the extended method. The above equations were derived from the following data.

England and Wales, 1910–12

Five-year period			Ten-year period		
Experience of age group	Observed death-rate	$_5p_x$	Experience of age group	Observed death-rate	$_{10}p_x$
15–20 M	.00288	.9861	25–35 M	.00480	.9531
20–25 F	.00314	.9845	30–40 F	.00514	.9498
5–10 F	.00318	.9835	30–40 M	.00614	.9403
5–10 M	.00320	.9832	35–45 F	.00652	.9365
25–30 F	.00367	.9819	35–45 M	.00799	.9224
20–25 M	.00372	.9814	40–50 F	.00838	.9186

It is evident that the observed death-rates are taken indiscriminately from male and female experiences and from different five- and ten-year age groups. The one necessary point is that r in the five-year group must lie about in the range (.00300 – .00370) and in the ten-year group about in the range (.00500 – .00800). This was found to be necessary because in different experiences wide variations in age x were found to correspond to the same value of r. Therefore it would be quite impossible to confine an observed death-rate to a fixed value of x, although n may be fixed as 5, 10, or any other desired term.

Having found $_np_x$ by this method for the age intervals 10–15, 15–20, 20–25, 25–35, . . . 75–85, it remains only to select a radix, say 10,000, at age 10, and to construct the l_x column for ages 10, 15, 20, 25, 25, . . . 85. For the end of the table log $_{10}p_{85}$ is assumed to be a linear function of the observed deat-hrate for the age interval 85 and upwards, and the resulting equation is

$$\log {}_{10}p_{85} = .188106 - 5.67829r,$$

which makes it possible to calculate l_{95}.

In order to calculate the functions L_x, T_x, in the stationary life table population, and the complete expectation of life, $\mathring{e}_x$, it is assumed that

$$l_x \cdot {}_nk_x = l_x + l_{x+1} + \quad . \quad . \quad . \quad + l_{x+n-1},$$

and that $_nk_x$ can be expressed as a linear or quadratic function of $_np_x$, or what amounts to the same thing, as a linear or quadratic function of r. The equation obtained for the five-year group is

$$k = 3.0914 + 1.9084p.$$

As in the case of expressing p in terms of r by a series of equations depending on the value of r, so a series of equations was determined for ${}_{10}k_x$ depending on the range of p. For example, when $.55 \leq {}_{10}p_x \leq .80$, then

$$ {}_{10}k_x = 14.4520 - .87293(3.2566 - p)^2. $$

For $x = 95$, ${}_{10}k_{95}$ was arbitrarily taken equal to 3. Having found the values of

$$ l_{10} \cdot {}_5k_{10} = l_{10} + l_{11} + l_{12} + l_{13} + l_{14}, $$
$$ l_{15} \cdot {}_5k_{15} = l_{15} + l_{16} + l_{17} + l_{18} + l_{19}, \text{ etc.}, $$

to the end of the table, the values of L_{10}, L_{15}, etc., and T_{10}, T_{15}, etc., are easily calculated. Then the complete expectation of life is derived from the relation $\mathring{e}_x = T_x / l_x$. The values found in this way for ages 10 and upwards are very close to the values in the extended tables and may be regarded as entirely satisfactory for the purposes of the registrar and health officer.

The treatment for ages under 10 is somewhat different. In fact, Dr. Snow divides the discussion into two main parts, ages under 10 corresponding approximately to falling or decreasing values of the death-rate r, and ages over 10 corresponding to rising or increasing values of the death-rate r. The treatment for ages under 10 is divided into four parts corresponding to the age intervals 5–10, 2–5, 1–2, 0–1. Linear equations are found for the 5–10 and 2–5 intervals connecting ${}_5p_5$ and ${}_3p_2$ respectively with r. Similarly for ${}_5k_5$, ${}_3k_2$, and p. For the interval 1–2 the relation between p_1 and r is hyperbolic:

$$ p_1 = .07434 + .92488 \left(\frac{2 - r}{2 + r} \right); $$

in other words, a linear relation is established between the chance of living a year and the observed central death-rate.

For the first year of life, age interval 0–1, p_0 is taken equal to the complement of the infantile mortality per birth, while

$$ k = \begin{cases} .7646 + .2354 \left(\dfrac{l_1}{l_0} \right) & \text{for males,} \\[2ex] .7871 + .2129 \left(\dfrac{l_1}{l_0} \right) & \text{for females.} \end{cases} $$

The proposed method is not novel but is rather a natural attempt to express mortality functions for arbitrary age intervals in terms of the observed death-rates by means of simple mathematical relations, namely, linear, quadratic, one to one hyperbolic. It is a suggestive line of thought, and a careful study of the report will well repay the student of vital statistics. To the actuary and mathematician it suggests further developments in the study of analytical relationships between the observed death-rate and the several functions appearing in the life table. It is not at all unlikely that these relations may be quite fully determined and placed on a more satisfactory mathematical foundation, as far as the theory and the proofs are concerned, than is indicated in this supplement. It is desirable to know not only the mathematical relation or equation connecting observed and computed functions, but also to be able to indicate the degree of approximation to which the computed function is obtained.

The explanation of the method is followed by ten pages of specially prepared numerical tables to facilitate the calculation of p from r and k from p, and a table of proportional parts. The remainder of the report includes sixty-five pages of tables, for males and for females, as follows: mean annual death-rates in several age periods in the areas for which life tables are given, 1911 and 1912; l_x, survivors at several ages per 100,000 born; $l_x - l_{x+n}$, deaths in several age periods per 100,000 born; $\mathring{e}_x$, complete expectation of life at several ages; survivors in the several areas as percentages of the survivors at the same age in England and Wales; expectations of life in the several areas as percentages of the expectations at the same age in England and Wales. The tables are presented on the whole in good form, except that in some cases, for example, on pages 31 and 63, the entire page is closely packed with figures without horizontal spaces every fifth or tenth row to relieve the eye and assist in following across the page. Such forbidding arrays of figures are not infrequent in the publications of this office, and the practice is much to be regretted.

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Foreign Exchange Before, During, and After the War. By T. E. Gregory, Cassel Reader in Commerce, University of London. Oxford University Press. 1921. Pp. 116.

The chief purpose of this excellently written little volume is evidently to restate in brief compass and in simple language the theory now associated with the name of its leading modern exponent, Professor Gustav Cassel. Little attempt is made at inductive verification. The statistical aspect of the problem, though referred to summarily in Chapter VI, "The Present Position of the Foreign Exchanges," is mostly relegated to a brief appendix of tables of exchange rates, prices, and paper money in the leading countries, which are not specifically related to the text.

As a restatement of the Cassel theory Mr. Gregory's essay accomplishes its purpose well. The first two chapters, on the exchange market and its meaning, and how to read the exchange article, are introductory, giving only enough of the technique to make the succeeding discussion of theory intelligible to the uninformed reader. The body of the book is in Chapters III to VI, where the writer discusses, in very broad outline only, the methods and forces governing exchange fluctuations under the conditions of a gold standard as before the war, the devices adopted and suggested for stabilizing the exchanges during the war, and the factors which govern the movements of "dislocated exchange" such as Europe has been experiencing. Following Cassel, he distinguishes between daily or short-time fluctuations which are but a reflection of market changes in the supply of and demand for bills, and the underlying movement of rates, which with Cassel he holds to be fundamentally determined by relative price levels and price changes in the trading countries.

Mr. Gregory insists upon dividing exchange experts into two camps, those who, like himself, think that the exchanges are dislocated *solely* by price inflation,